**Definition**: Theoretical probability refers to situations where the set of all equally likely outcomes is known. Any subset of these outcomes is known as an event.

**Example**: Suppose you are flipping a coin. Probability refers to the chance of getting one event (Heads) over another equally likely event (Tails). Notationally, most list outcomes of the form \{H, T\}

To compute the probability, you need to know the number of ways and event can occur, and you also need to know the total possible outcomes.

**Example**: In flipping a coin, there are two possible outcomes \{H, T\}. The probability you will get heads is 1/2, or 50%.

**Example (Checkpoint 1)**: A die is rolled once. Find the probability of rolling
a) a 2
b) a number less than 4
c) a number greater than 7
d) a number less than 7

**Solution**: A die has 6 sides, so there are 6 possible outcomes \{1,2,3,4,5,6\}
a) rolling a 2 can happen 1 way \(\Rightarrow\) \(1/6 = 16.7\%
 b) rolling a number less than 4 can happen 3 ways \{1,2,3\} \(\Rightarrow\) \(3/6 = 1/2 = 50\%
c) there is no possible way to get a number larger than 7 \(\Rightarrow\) \(0/6 = 0\%
d) rolling a number less than 6 can happen 6 ways \(\Rightarrow\) \(6/6 = 1 =100\%

**Question**: What do we know about the value of probability?

**Answer**: It will always be between 0 and 1 (or 0% and 100%). You cannot have a negative probability or a probability greater than 1.

**Example (Checkpoint 2)**: You are dealt one card from a standard 52 card deck. Find the probability of being dealt
a) an ace
b) a red card
c) a red king

**Solution**: If you are not familiar with a deck of cards, refer to the figure on page 603 of the text.
a) 4 aces \(\Rightarrow\) \(4/52 = 1/13\)
b) 13 hearts and 13 diamonds = 26 red cards \(\Rightarrow\) \(26/52 = 1/2\)
c) 2 red kings \(\Rightarrow\) \(2/52 = 1/26\)
Example: Each person carries two genes that determine eye color. One of these genes (bey2) has two flavors (or possibilities), brown or blue with brown being dominant. Describe the ways these two can be combined, and what is the probability of having bey2 gene that produces blue eyes?

Solution: Let B = brown and b = blue

One could have BB, Bb, bB or bb for a bey2 gene

Since blue is recessive, the only way to get blue is with bb $\Rightarrow 1/4 = 25\%$ chance

Definition: Empirical Probability refers to situations where we observe the frequency of occurrences of events, it may not actually be known or predictable. The empirical probability of an event $E$ is given by

$$P(E) = \frac{\text{observed number of times } E \text{ occurs}}{\text{total number of observations}}$$

Example (Checkpoint 4): Using the chart on page 606 for Arab-American Faiths, if an Arab American is selected at random, find the empirical probability of selecting a Muslim.

Solution: Adding up all the possibilities, we have 3 million Arab-American faiths

There are 0.69 million Muslims $\Rightarrow 0.69 \text{ mill} / 3 \text{ mill} = 23\%$